TABE

MATH - D

Unit - 1

Lesson - 10

Rational Numbers
COMPARE

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Estimate and Compare Irrational Numbers

You can estimate the value of irrational numbers using approximations. For example, pi (π) can be approximated as different numbers depending on how it is rounded. Pi (π) is often approximated as 3, 3.14, or 3.142.

Example If $\pi \approx 3$, what is the approximate value of $\pi - 2$?

1) Substitute 3 for π. $\pi - 2 \approx 3 - 2$

2) Subtract. $3 - 2 = 1$, so $\pi - 2 \approx 1$

You can also approximate the square root of non-perfect squares by determining which perfect squares are close to the non-perfect square.

Example Dante calculates that the distance from one corner of his kitchen table to the opposite corner is $\sqrt{80}$ feet. Plot a point that represents the distance on a number line.

1) Determine the two perfect squares between which $\sqrt{80}$ lies.

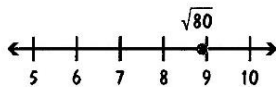 $8^2 = 64$, so $\sqrt{64} = 8$ and $9^2 = 81$, so $\sqrt{81} = 9$

   Therefore, $\sqrt{80}$ is between 8 and 9.

2) Determine whether $\sqrt{80}$ is closer to 8 or 9.

   Since 80 is closer to 81 than to 64, $\sqrt{80}$ will be very close to 9.

3) Plot the approximation of $\sqrt{80}$ on the number line.
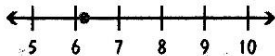
   $\sqrt{80}$ is located just before 9.

> **Hint**
> A perfect square is the product of a rational number multiplied by itself. For example, 3^2 is a perfect square.

1. Which irrational number is shown on the number line?

> **Strategy**
> Square the integers that the point is between. Then, determine which non-perfect square would lie between those two perfect squares.

A. $\sqrt{30}$

B. $\sqrt{38}$

C. $\sqrt{46}$

D. $\sqrt{51}$

1. **B** $6^2 = 36$ and $7^2 = 49$, so $\sqrt{38}$ will be between 6 and 7 but much closer to 6.

Read each question. Select the correct answer.

1 Where would $\sqrt{20}$ be located on a number line?

A. between 2 and 3

B. between 3 and 4

C. between 4 and 5

D. between 5 and 6

2 Where would $\sqrt{56}$ be located on a number line?

A. between 7 and 8

B. between 8 and 9

C. between 9 and 10

D. between 10 and 11

3 Where would $\sqrt{95}$ be located on a number line?

A. between 7 and 8

B. between 8 and 9

C. between 9 and 10

D. between 10 and 11

4 Where would $\sqrt{39}$ be located on a number line?

A. between 3 and 4

B. between 4 and 5

C. between 5 and 6

D. between 6 and 7

5 If $\pi \approx 3.14$, which of the following statements is true?

A. $\pi^2 > 9$

B. $2\pi = 6$

C. $\pi + 4 < 7$

D. $-\frac{\pi}{2} > -1.5$

6 Which of the following statements is true?

A. $\sqrt{38} < 6$

B. $\sqrt{110} = 10^2$

C. $7 > \sqrt{58}$

D. $11 < \sqrt{125}$

7 Which of the following statements is true?

A. $\sqrt{45} < 6$

B. $\sqrt{87} > 3^2$

C. $5 > \sqrt{35}$

D. $10 > \sqrt{114}$

8 Order the numbers from GREATEST to LEAST.

A. $\sqrt{36}, \sqrt{49}, \sqrt{42}, \sqrt{52}$

B. $\sqrt{42}, \sqrt{49}, \sqrt{52}, \sqrt{36}$

C. $\sqrt{52}, \sqrt{49}, \sqrt{42}, \sqrt{36}$

D. $\sqrt{36}, \sqrt{42}, \sqrt{49}, \sqrt{52}$

9 Which number is closest to $\sqrt{79}$?

A. 8

B. 8.3

C. 8.5

D. 9

10 If $\pi \approx 3$, which numbers are arranged from LEAST to GREATEST?

A. $(5.2 - \pi), \frac{\pi}{3}, 3\pi, (\pi + 3)$

B. $\frac{\pi}{3}, (5.2 - \pi), (\pi + 3), 3\pi$

C. $\frac{\pi}{3}, 3\pi, (\pi + 3), (5.2 - \pi)$

D. $3\pi, (\pi + 3), (5.2 - \pi), \frac{\pi}{3}$

11 If $\pi \approx 3$, what is the approximate value of -5π?

A. -53

B. -15

C. -8

D. -2

12 If $\pi \approx 3$, what is the approximate value of $\sqrt{11} + 5\pi$?
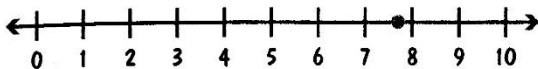
A. 23   B. 20

C. 18   D. 15

13 Which irrational number is graphed on the number line?

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←+—+—+—+—+●+—+—+—+—+—+→
  0   1   2   3   4   5   6   7   8   9   10
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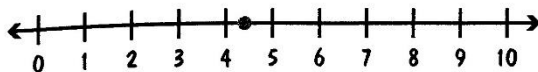
A. $\sqrt{15}$

B. $\sqrt{19}$

C. $\sqrt{24}$

D. $\sqrt{26}$

14 Which irrational number is graphed on the number line?

~~~
←+—+—+—+—+—+—+—+●+—+—+→
  0   1   2   3   4   5   6   7   8   9   10
~~~

A. $\sqrt{60}$

B. $\sqrt{64}$

C. $\sqrt{70}$

D. $\sqrt{72}$

15 Which irrational number is graphed on the number line?

~~~
←+—+●+—+—+—+—+—+—+—+—+→
  0   1   2   3   4   5   6   7   8   9   10
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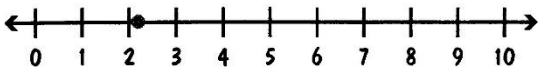
A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\sqrt{4}$

D. $\sqrt{5}$

16 Which irrational number is graphed on the number line?

~~~
←+—+—+—+—+●+—+—+—+—+—+→
  0   1   2   3   4   5   6   7   8   9   10
~~~

A. $\sqrt{15}$

B. $\sqrt{18}$

C. $\sqrt{24}$

D. $\sqrt{27}$

1 Roman calculates the height of a projectile after six seconds in the air to be $\sqrt{50}$ meters. Where is this height located on a number line?

A. between 5 and 6

B. between 6 and 7

C. between 7 and 8

D. between 8 and 9

2 Joanna has a glass cylinder with a circumference of 11π centimeters. If $\pi \approx 3.14$, what is the approximate circumference?

A. 11 cm   B. 14 cm

C. 27 cm   D. 35 cm

3 An engineer calculates the time it takes for a falling object to travel a certain distance. He graphs the time on a number line. What time is shown?
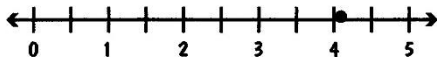
A. $\sqrt{17}$ s   B. $\sqrt{24}$ s

C. $\sqrt{14}$ s   D. $\sqrt{8}$ s

4 Kate uses the length and height of her television to calculate the length of the diagonal. Which measurement is closest to the diagonal's length of $\sqrt{14}$ feet?

A. 2.5 ft   B. 3.1 ft

C. 3.7 ft   D. 4.2 ft

5 Based on the area, Alex finds the approximate length of his property in the city to be eight meters. Which measurement is closest to the approximate length of Alex's property?

A. $\sqrt{16}$ m   B. $\sqrt{48}$ m

C. $\sqrt{62}$ m   D. $\sqrt{82}$ m

6 The area of a circular room is graphed on the number line. What is the room's area?

A. 20π ft^2 B. 25π ft^2
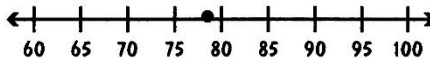
C. 30π ft^2 D. 35π ft^2

7 Tina calculates the time it takes for a particular comet to orbit the sun is $\sqrt{95}$ months. Where is this time located on a number line?

A. between 7 and 8

B. between 8 and 9

C. between 9 and 10

D. between 10 and 11

8 If $\pi \approx 3.14$, which inequality is true?

A. $4\pi - 4 < 11$

B. $5\pi - 5 > 16$

C. $3\pi - 5 < 4$

D. $2\pi - 2 > 5$

9 Charlie calculates the revenue of his business to be $\sqrt{67}$ million dollars. Which amount is closest to his revenue?

A. $6 million   B. $7 million

C. $8 million   D. $9 million

10 Based on an equation for the deceleration after braking in his car, Kevin calculates that it will take about 5.1 seconds to stop when driving. Which irrational number is closest to this approximation?

A. $\sqrt{24}$ s   B. $\sqrt{26}$ s

C. $\sqrt{34}$ s   D. $\sqrt{37}$ s

11 The distance between an atom's nucleus and its first electron in nanometers is graphed on the number line. What distance is shown?
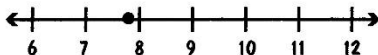
A. $\sqrt{48}$ nm   B. $\sqrt{51}$ nm

C. $\sqrt{61}$ nm   D. $\sqrt{65}$ nm

12 Miranda is programming a drone and calculates the distance of the flight to be $\sqrt{23}$ kilometers. Which is closest to the distance?

A. 3.9 km   B. 4.1 km

C. 4.8 km   D. 5.2 km

13 The approximate time it will take Stephen's business to become profitable is 3.9 years. Which time is closest to this approximation?

A. $\sqrt{15}$ yr   B. $\sqrt{17}$ yr

C. $\sqrt{22}$ yr   D. $\sqrt{26}$ yr

14 Wendy calculates the height of her weather balloon to be $\sqrt{29}$ kilometers. Where is the height located on a number line?

A. between 3 and 4

B. between 4 and 5

C. between 5 and 6

D. between 6 and 7

15 A machine uses steel bearings with a radius between 11π and 12π millimeters. If $\pi \approx 3.14$, which two measurements fall within this range?

A. 32 mm

B. 33 mm

C. 35 mm

D. 36 mm

E. 39 mm

F. 40 mm

16 The distance from the outside edge of a building to its center is graphed. Which distance is shown?
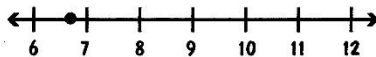
A. $\sqrt{37}$ yd   B. $\sqrt{45}$ yd

C. $\sqrt{50}$ yd   D. $\sqrt{68}$ yd

17 Clifton has a board that is $\sqrt{11}$ feet long. Which measurement is closest to the board's approximate length?

A. $3\frac{5}{16}$ ft

B. $3\frac{13}{16}$ ft

C. $4\frac{1}{8}$ ft

D. $4\frac{9}{16}$ ft

18 Erika is painting a spherical roof of a structure and calculates the surface area to be 9π square meters. If $\pi \approx 3.14$, where would this area be graphed on a number line?

A. between 25 and 27

B. between 27 and 29

C. between 29 and 31

D. between 31 and 33

19 Fred calculates the distance from his home to his work to be about 7.7 miles. What distance is closest to this approximation?

A. $\sqrt{42}$ mi   B. $\sqrt{47}$ mi

C. $\sqrt{51}$ mi   D. $\sqrt{59}$ mi

20 If $\pi \approx 3.14$, which inequality is true?

A. $2\pi + 3 > 15$

B. $4\pi - 5 > 11$

C. $3\pi - 6 < 10$

D. $5\pi + 7 < 20$

Math-D Lesson-10 Key

Lesson 10 | **Estimate and Compare Irrational Numbers**

1. C. Since 20 is between 16 (4^2) and 25 (5^2), $\sqrt{20}$ is between 4 and 5.

2. A. Since 56 is between 49 (7^2) and 64 (8^2), $\sqrt{56}$ is between 7 and 8 .

3. C. Since 95 is between 81 (9^2) and 100 (10^2), $\sqrt{95}$ is between 9 and 10.

4. D. Since 39 is between 36 (6^2) and 49 (7^2), $\sqrt{39}$ is between 6 and 7.

5. A. Because π is a little greater than 3, π^2 is a little greater than 3^2, or 9.

6. D. Because $11^2 = 121$, 11 is less than $\sqrt{125}$.

7. B. Because $9^2 = 81$, $\sqrt{87} > 9$, or 3^2.

8. C. The square roots of numbers listed in decreasing order are themselves in decreasing order.

9. D. $\sqrt{79}$ is between 8 and 9. $8^2 = 64$. $9^2 = 81$. Since you know $\sqrt{79}$ is closer to 9, you narrow down the possible answers to 8.5 and 9. $8.5^2 = 72.25$, so $\sqrt{79}$ is closer to 9.

10. B. Find the approximate value of each expression by substituting 3 for π. $\frac{\pi}{3} = 1$, $(5.2 - \pi) = 2.2$, $(\pi + 3) = 6$, $3\pi = 9$.

11. B. To find an approximate value of -5π, substitute 3 for π. $-5\pi = -5(3) = -15$.

12. C. To find an approximate value of $\sqrt{11} + 5\pi$, approximate the value of $\sqrt{11}$. $3^2 = 9$. $4^2 = 16$, so $\sqrt{11}$ is closer to 3. Substitute 3 for $\sqrt{11}$ and for π. $\sqrt{11} + 5\pi = 3 + 5(3) = 18$.

13. B. $\sqrt{19}$ is between 4 and 5, but closer to 4.

14. A. $\sqrt{60}$ is between 7 and 8, but closer to 8.

15. D. $\sqrt{5}$ is between 2 and 3, but much closer to 2.

16. C. $\sqrt{24}$ is between 4 and 5, but much closer to 5.

Math-D   Practice-10 Key

1. C. Because 50 is between 49 (7^2) and 64 (8^2), $\sqrt{50}$ is between 7 and 8 on a number line.

2. D. Because π is a little greater than 3, the circumference is greater than 33 cm.

3. A. $\sqrt{17}$ is between 4 and 5, but closer to 4.

4. C. $\sqrt{14}$ is closer to 4 than it is to 3, but it is less than 4.

5. C. 62 is closest to 64 (8^2).

6. B. Because π is a little greater than 3, 25π is greater than 75.

7. C. Because 95 is between 81 (9^2) and 100 (10^2), $\sqrt{95}$ is between 9 and 10 on a number line.

8. A. Because π is a little greater than 3, $4\pi - 4$ is a little greater than 8, which is less than 11.

9. C. Because 67 is between 64 (8^2) and 81 (9^2), $\sqrt{67}$ is between 8 and 9. Because 67 is only three units away from 64, 8 million dollars is a closer approximation of the revenue.

10. B. Because 5.1 is a little greater than 5, it is also a little greater than $\sqrt{25}$ which is equivalent to 5.

11. C. Because 61 is between 49 (7^2) and 64 (8^2), $\sqrt{61}$ is between 7 and 8 on a number line. The distance is closer to 8 nanometers, because 61 is only three units away from 64.

12. C. Because 23 is less than 25 (5^2), the distance is a little less than 5 kilometers.

13. A. 15 is closest to, but a little less than, 16 (4^2).

14. C. Because 29 is between 25 (5^2) and 36 (6^2), $\sqrt{29}$ is between 5 and 6 on a number line.

15. C, D. 11π is more than 33, and 12π is more than 36. The two numbers that fall within this range are 35 and 36.

16. B. Because 45 is between 36 (6^2) and 49 (7^2), $\sqrt{45}$ is between 6 and 7 on the number line. The distance is closer to 7 yards because 45 is only four units away from 49.

17. A. Because 11 is between 9 (3^2) and 16 (4^2), $\sqrt{11}$ is between 3 and 4. The board's length is closer to 3 feet because 11 is only two units away from 9.

18. B. Because π is a little more than 3, 9π is greater than 27, so it is between 27 and 29 on a number line.

19. D. 59 is closest to, but a little less than, 64 (8^2).

20. C. Because π is a little greater than 3, $3\pi - 6$ is a little greater than 3, which is less than 10.